HEAT TRANSFER DURING LAMINAR FLUID FLOW IN A PIPE WITH RADIATIVE HEAT REMOVAL

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The heat-transfer problem is analyzed for laminar fluid flow in the initial section of a circular pipe having a parabolic entry velocity distribution and heat removal by radiation from the surface of the pipe. Analytical relations are derived for the criterion Nu as a function of the longitudinal coordinate and for its limiting value Nu_∞ as a function of the radiation parameter Φ.

The heat-transfer problems associated with laminar fluid flow in a pipe for a given wall temperature or heat flux have been investigated in detail in [1-3]. There are several problems, however, in which the boundary conditions prove to be more complicated, representing in general a certain relationship between the wall temperature and heat flux, as well as the parameters of the external medium. A specific example of such a problem is radiative heat removal from a pipe in which a liquid to be cooled is flowing. In the case of an isolated pipe radiating into a diathermal medium in the absence of convective and conductive heat removal the heat flux is proportional to the fourth power of the pipe wall temperature. The stated problem has been investigated in [6]. In the present article we solve the problem by a simpler method, reducing it to the solution of a set of ordinary differential equations.

Let us consider a flow of an incompressible fluid in a circular pipe in the presence of hydrodynamic stabilization. We assume that the physical attributes of the fluid are invariant and the entry temperature is constant over the cross section. Heat removal takes place from the pipe surface due to radiation, and the temperature of the ambient medium is zero. With hydrodynamic stabilization the radial distribution of the velocity obeys the Poiseuille law

$$u = 2\bar{u}\left[1 - 4\left(\frac{r}{d}\right)^2\right],$$

and the energy equation has the form

$$2\rho c_p\bar{u}\left[1 - 4\left(\frac{r}{d}\right)^2\right]\frac{\partial T}{\partial x} = \frac{\lambda}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

or, since $\lambda/\bar{\rho}uc_p = d/\mathrm{Pe}$,

$$2\left[1 - 4\left(\frac{r}{d}\right)^2\right]\frac{\partial T}{\partial x} = \frac{d}{\mathrm{Pe}}\cdot\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

with the boundary and initial conditions

$$r = 0 \quad \partial T/\partial r = 0,$$

$$r = d/2 \quad \lambda\partial T/\partial r = q = -\varepsilon\sigma T_w^4,$$

$$x = 0 \quad T = T_0, \quad T_w = T_0.$$

We introduce $X = x/d\mathrm{Pe}$; $r' = 2r/d$; $T' = T/T_0$; $q' = q/q_0$; T_W'/T_0 and, dropping the prime from the variables, we write the energy equation in the form

$$(1 - r^2)\frac{\partial T}{\partial X} = \frac{2}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{1}$$

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with the boundary and initial conditions

$$r=0 \quad \frac{\partial T}{\partial r}=0, \tag{2a}$$

$$r=1 \quad \frac{\partial T}{\partial r}=q=-\Phi T_w^4, \tag{2b}$$

$$X=0 \quad T=1, \ \ T_w=1. \tag{2c}$$

We solve Eq. (1) by a method representing a generalization of the method described in [5] to the case of an incompressible fluid flow in a circular pipe. We solve the equation for the leading derivative and intergrate term-by-term over the radius:

$$r\frac{\partial T}{\partial r}=\frac{1}{2}\int_0^r \frac{\partial T}{\partial X}(1-r^2)\,rdr+C_1(X), \tag{3}$$

where $C_1(X) = 0$ due to condition (2a). We then divide both sides of the new equation by r and again integrate over the radius:

$$T=T_e(X)+\frac{1}{2}\int_0^r \frac{dr}{r}\int_0^r \frac{dT}{dX}(1-r^2)\,rdr. \tag{4}$$

As a first approximation we represent the expression for the temperature profile in the form

$$T=a(X)+b(X)\,r^{\alpha(X)}$$

or, complying with conditions (2b) and (2c),

$$T=T_w+\frac{q}{\alpha}(r^{\alpha}-1). \tag{5}$$

We substitute this approximate relation into the right-hand side of Eq. (4) and take two quadratures. We then obtain the second-approximation temperature profile:

$$T=T_e(X)+\frac{1}{2}\left\{\frac{da}{dX}\left(\frac{r^2}{4}-\frac{r^4}{16}\right)+\left(\frac{1}{\alpha}\cdot\frac{dq}{dX}-\frac{qd\alpha}{\alpha^2dX}\right)\right.$$
$$\left.\times\left(\frac{r^{\alpha+2}}{(\alpha+2)^2}-\frac{r^{\alpha+4}}{(\alpha+4)^2}\right)+\frac{q}{\alpha}\cdot\frac{d\alpha}{dX}\left[\frac{r^{\alpha+2}}{(\alpha+2)^2}\left(\ln r-\frac{2}{(\alpha+4)^2}\right)\left(\ln r-\frac{2}{\alpha+4}\right)\right]\right\}. \tag{6}$$

In the method described in [5] the second approximation for the desired function depends only on one unknown parameter, and by satisfying one of the boundary conditions it is possible to obtain an ordinary differential equation for the variation of this parameter along the longitudinal coordinate. In our case, to find $T_W(X)$ and $\alpha(X)$ we substitute expression (5) into Eqs. (3) and (4) and set the limits of integration in these equations equal to unity. Then after integration and a few straightforward transformations we obtain

$$\frac{d}{dX}\left[\frac{(3\alpha+10)T_w^4}{(\alpha+2)^2(\alpha+4)^2}\right]=T_w^4\left(3-\frac{4}{\alpha}\right), \tag{7}$$
$$\frac{d}{dX}\left[\frac{T_w}{4\Phi}+\frac{(\alpha+6)\,T_w^4}{8(\alpha+2)(\alpha+4)}\right]=-T_w^4$$

with the following initial conditions: $\alpha \to \infty$ and $T_W = 1$ for $X = 0$, as implied by relation (2c). To find the unknown function $T_e(X)$ entering into (6) we use the relation for the mass-average flow temperature. We write the average temperature in the form

$$\bar{T}=2\int_0^1 2(1-r^2)\,Trdr. \tag{8}$$

Then from Eq. (1), using condition (2b), we obtain

$$2\int_0^1 2(1-r^2)\frac{\partial T}{\partial X}rdr=8\int_0^1 \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)dr$$

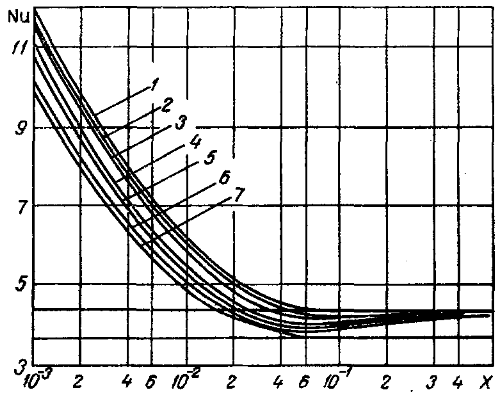

Fig. 1. Criterion Nu versus X for $T_k = 0$. 1) q = const; 2) $\Phi = 0.2$; 3) 1; 4) 5; 5) 10; 6) 50; 7) T_w = const.

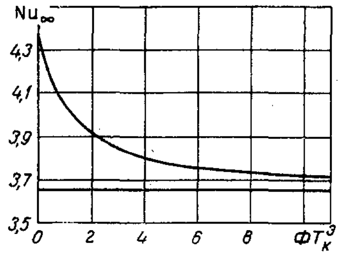

Fig. 2. Limiting criterion Nu_∞ versus parameter ΦT_k^3.

and

$$\bar{T} = 1 + 8 \int_0^X q dX. \tag{9}$$

Condition (2c) renders the constant of integration in the latter equation equal to unity.

Next we substitute expression (6) into Eq. (8), carry out the integration and, using the relationship between the resulting value of the mass-average temperature and expression (9), find the unknown function $T_e(X)$:

$$T_e = \bar{T} + \frac{1}{2}\left\{-\frac{7}{96}\left[\frac{dT_w}{dX} - \frac{d}{dX}\left(\frac{q}{\alpha}\right)\right] - \frac{d}{dX}\left(\frac{q}{\alpha}\right) \times \frac{32(2\alpha+7)}{(\alpha+2)^2(\alpha+4)^2(\alpha+6)(\alpha+8)} - \frac{16q}{\alpha(\alpha+4)^2(\alpha+6)} \times \left[\frac{2\alpha^2+25\alpha+76}{(\alpha+4)(\alpha+8)^2} - \frac{2\alpha^2+17\alpha+34}{(\alpha+2)^2}\right]\frac{d\alpha}{dX}\right\}. \tag{10}$$

Consequently, by inserting expression (10) into (6) we finally obtain the temperature profile:

$$T = 1 + 8\int_0^X q dX + \frac{1}{2}\left\{\left[\frac{dT_w}{dX} - \frac{d}{dX}\left(\frac{q}{\alpha}\right)\right]\left(\frac{r^2}{4} - \frac{r^4}{16} - \frac{7}{96}\right) + \left(\frac{r^{\alpha+2}}{(\alpha+2)^2} - \frac{r^{\alpha+4}}{(\alpha+4)^2} - \frac{32(2\alpha+7)}{(\alpha+2)^2(\alpha+4)^2(\alpha+6)(\alpha+8)}\right)\frac{d}{dX}\left(\frac{q}{\alpha}\right) + \frac{q}{\alpha}\cdot\frac{d\alpha}{dX}\left[\frac{r^{\alpha+2}}{(\alpha+2)^2}\left(\ln r - \frac{2}{\alpha+2}\right) - \frac{r^{\alpha+4}}{(\alpha+4)^2}\left(\ln r - \frac{2}{\alpha+4}\right) - \frac{16}{(\alpha+4)^2(\alpha+6)}\left(\frac{2\alpha^2+25\alpha+76}{(\alpha+4)(\alpha+8)^2} - \frac{2\alpha^2+17\alpha+34}{(\alpha+2)^3}\right)\right]\right\}. \tag{11}$$

We can further refine the temperature profile by using expression (11) as the first approximation. It is important to note that the substance of our ensuing discussion remains unchanged in this case, other than the fact that the second derivatives evolving for the function $T_w(X)$ and $\alpha(X)$ upon substitution of (11) into the right-hand side of Eq. (4) can be obtained by differentiation of the equations in the system (7). Experience has shown, however, that further iterations are unnecessary, because the accuracy with which expression (11) describes the temperature profile (1.5% error) is acceptable for engineering calculations. Now the dependence of the criterion Nu on the coordinate X can be written as follows:

$$\mathrm{Nu} = 48 \Big/ \Big[11 - \frac{11\alpha+106}{(\alpha+2)(\alpha+4)(\alpha+6)(\alpha+8)} \cdot \frac{1}{q} \cdot \frac{dq}{dX} - \left(\frac{3\alpha-4}{\alpha} - \frac{1}{q}\cdot\frac{dq}{dX}\cdot\frac{3\alpha+10}{(\alpha+2)^2(\alpha+4)^2}\right) \tag{12}$$

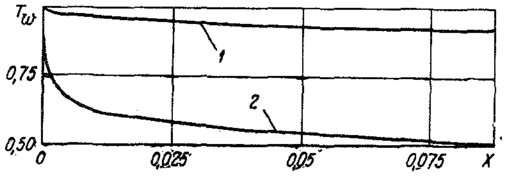

Fig. 3. Wall temperature T_W versus X for $T_k = 0$. 1) $\Phi = 0.2$; 2) 10.

$$\times \frac{(\alpha+2)(\alpha+4)(33\alpha^4+864\alpha^3+7900\alpha^2+29680\alpha+38176)}{(9\alpha^2+58\alpha+96)(\alpha+6)^2(\alpha+8)^2}\Bigg].$$

Here $1/q \cdot dq/dX = 4/T_W \cdot dT_W/dX$, and T_W α are found from the solution of the set of ordinary differential equations (7). It is important to realize that in comparing expression (12) with the exact solution according to [2] for the case q = const the discrepancy is 1 to 1.5%. It was assumed in the derivation of condition (2c) that the heat-transfer coefficient in the initial cross section tends to infinity, even through the problem does not exclude the admissibility of specifying a temperature jump between the fluid and wall at the pipe entry.

We solved the set of equations (7) numerically. The results of the calculations for the local criterion Nu are given in Fig. 1. Curves 1 and 7 were plotted for the cases of a constant heat flux and constant wall temperature, respectively, and curves 2, 3, 4, 5, and 6 were plotted for the following values of the parameter Φ: 0.2, 1, 5, 10, and 50. A comparison with the results of [6] yields a discrepancy of 1 to 1.5%. As $\Phi \to 0$ and $\Phi \to \infty$ the set of equations (7) degenerates into the set of equations corresponding to specification of a constant heat flux at the pipe wall and a constant pipe wall temperature, respectively. If the temperature of the surrounding medium is not equal to zero, we have $q = -\Phi(T_W^4 - T_k^4)$. We solve the system (7) for $d\alpha/dX$ and $(1/q)(dq/dX)$, substituting $(1/q)(dq/dX)$ into (12) and putting $d\alpha/dX = 0$ for $X \to \infty$. We obtain the following expression for the limiting value Nu_∞ of the criterion:

$$Nu_\infty = 48 \Bigg/ \left[11 - \frac{(11\alpha_\infty+106)(3\alpha_\infty-4)(\alpha_\infty+2)(\alpha_\infty+4)}{\alpha_\infty(\alpha_\infty+6)(\alpha_\infty+8)(3\alpha_\infty+10)}\right],$$

in which α_∞ is determined from the equation

$$8 + \frac{(3\alpha_\infty-4)(\alpha_\infty+2)^2(\alpha_\infty+4)^2}{\alpha_\infty(3\alpha_\infty+10)}\left[\frac{1}{2\Phi T_k^3} + \frac{\alpha_\infty+6}{(\alpha_\infty+2)(\alpha_\infty+4)}\right] = 0.$$

The dependence of the limiting criterion Nu_∞ on the parameter ΦT_k^3 is shown in Fig. 2. The values of the local criterion Nu for various values of the parameter Φ when the temperature of the external medium is equal to zero is described by the following interpolation formula (based on the values of the parameter in Fig. 1):

$$\frac{Nu}{Nu_{g=\mathrm{const}}} = 0.94 - \frac{0.0061 - 0.0053\ln X}{1 + 0.0242\ln X}\ln\Phi$$

with 1 to 2% error in the range $0.001 < X < 0.2$ and $0.1 < \Phi < 50$. The results of calculations of the limiting criterion Nu_∞ are shown in Fig. 2, where they are described by the interpolation formula

$$Nu_\infty = \frac{4.364 + 3.66\Phi T_k^3}{1 + \Phi T_k^3}$$

with 0.5% error.

The dependence of the second-approximation pipe wall temperature on X is given in Fig. 3 for values of the parameter $\Phi = 0.2$ and 10 (curves 1 and 2, respectively). A sharp drop in the pipe wall temperature is observed in the initial section.

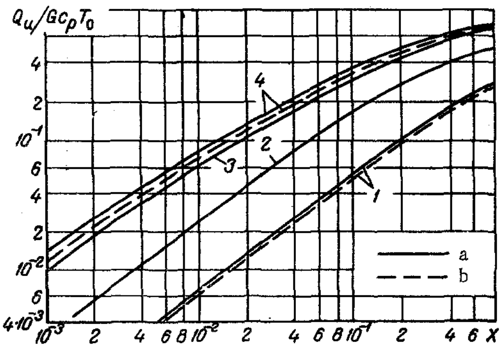

Fig. 4. Dependence of heat discharged on X for $T_k = 0$: a) calculation for $q \sim T_W^4$ by means of Nu; b) the same, for T_W = constant. 1) $\Phi = 0.2$; 2) 1; 3) 5; 4) 10.

The dependence of Q_u/Gc_pT_0 on X is shown in Fig. 4. Curves 1 through 4 were plotted for values of $\Phi = 0.2$, 1, 5, and 10, respectively. Also shown in this figure for comparison are alternative curves 1 and 4 plotted for the case of a constant pipe wall temperature for values of the parameter $\Phi = 0.2$ and 10. It is evident from the graph that the difference for the neglected heat flux for the corresponding curves runs from 10 to 1% in the interval $0.001 < X < 0.1$.

NOTATION

x, r	are the axial and radial coordinates;
d	is the pipe diameter;
T, T_0, T_w, T_k	are the temperatures of the fluid, the fluid at entry to the pipe, the pipe wall, and the surrounding medium;
u	is the fluid velocity in the pipe;
$\bar{u}$	is the average fluid velocity;
ρ, c_p, λ	are the density, heat capacity, and thermal conductivity of the fluid;
ε	is the emissivity of the pipe surface;
σ	is the Stefan–Boltzmann constant;
q, q_0	is the heat flux at the wall and entry of the pipe;
$\bar{T}$	is the average fluid temperature;
G	is the volumetric flow rate of fluid;
$X = x/dPe$	is the reduced axial coordinate;
Pe	is the Peclet number;
$Nu = 2a/(T_w - T)$	is the Nusselt number;
Nu_∞	is the limiting value of the Nusselt number;
$\Phi = \varepsilon\sigma T_0^3/2\lambda$	is the radiation parameter;
$Q_4 = \quad qdX$	is the integral heat flux from pipe surface.

LITERATURE CITED

1. W. Nusselt, Z. VDI, 54, 1154-1158 (1910).
2. R. Siegel, E. M. Sparrow, and T. M. Hallman, Appl. Sci. Res., 7, No. 5 (1958).
3. B. S. Petukhov, Heat Transfer and Drag for Laminar Fluid Flow in Pipes [in Russian], Énergiya (1967).
4. L. S. Leibenzon, Trudy TsAGI, 240 (1935).
5. S. M. Targ, Fundamental Problems of Laminar Flow Theory [in Russian], Gostekhizdat (1951).
6. J. C. Chen, Internat. J. Heat and Mass Trans., 9 (May, 1966).